

A Probabilistic Approach for Mine Burial Prediction

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ABSTRACT

Predicting the degree of burial of mines in soft sediments is one of the main concerns of Naval Mine CounterMeasures (MCM) operations. This is a difficult problem to solve due to uncertainties and variability of the sediment parameters (i.e., density and shear strength) and of the mine condition at contact with the seafloor (i.e., vertical and horizontal velocity, angular rotation rate, and pitch angle at the mudline). A stochastic approach is proposed in this paper to better incorporate the dynamic nature of free-falling cylindrical mines in the modeling of impact burial. The orientation, trajectory and velocity of cylindrical mines, after about 4 meters free-fall in the water column, are very strongly influenced by boundary layer effects causing quite chaotic behavior. The model's convolution of the uncertainty through its nonlinearity is addressed by employing Monte Carlo simulations. Finally a risk analysis based on the probability of encountering an undetectable mine is performed.

Keywords: Mine burial, mine detection, Monte Carlo simulation.

1. INTRODUCTION

Increased mine countermeasures (MCM) capability is listed as a U.S. Navy Fleet Required Capability to identify and focus experimental efforts and resources to achieve Sea Power-21 goals¹. The detection, location and classification of buried mines is a particularly difficult problem². This is a difficult task due to uncertainties and variability of the environment (i.e., density and bearing strength of the sediment) and of the mine condition at contact with the seafloor (i.e., vertical and horizontal velocity, angular rotation rate, and pitch angle at the mudline).

The Monte Carlo algorithm was used to develop an understanding of the stochastic nature of the burial depth to be expected for one mine shape in one representative seafloor. The variability/uncertainty of the input parameters, i.e., mine shape dynamics and sediment properties, can be expressed by their probability density functions (pdf's). These pdf's describe the state of the overall stochastic system. The goal of the Monte Carlo method is to simulate the physical system by random sampling from these pdf's and by performing the necessary supplementary computations needed to describe the response of the system. Thus, the results of model runs can be used to build the frequency histograms for various output parameters (e.g., mine surface area and volume buried, mine burial depth, and pitch angle at rest in seabed). Mine CounterMeasures (MCM) operations are mainly concerned about how many mines may be undetectable due to their burial in sediments. One

can compute the probability of not detecting (the risk of not detecting) buried mines given the critical detection limit, based on the histograms generated by the Monte Carlo simulations.

2. RELATED WORK

The first physics-based mine impact burial model was documented by Arnone and Bowen² in 1980. This model has been refined by several researchers such as Satkowiak³, Hurst⁸, and Mulhern⁴ and a sensitivity study of the model reported by Chu et al.⁵. The model is of a deterministic type, for which all the input parameters (environmental physical parameters, mine geometry and mine deployment conditions) should be known a priori. However, the real world conditions make the mine burial prediction problem more complex due to uncertainties and variability of the environment (i.e., density and bearing strength of the sediment layers). Even more so, the real behavior of a mine shape in free-fall in the water column is not deterministic, as assumed in the model, but highly random in nature⁶. In reality, the hydrodynamic forcing acting on cylindrical mine shapes, after falling approximately through 4 m of water, is governed by boundary layer effects, and the trajectory becomes quite chaotic⁷. At first contact with the seafloor, the mine shape will exhibit a wide distribution of conditions (i.e., vertical and horizontal velocities, angular rotation rate, pitch angle, etc.).

Stochastic approaches have been invoked recently to deal with uncertainty and variability of the input data. Rennie and Brandt⁶ developed an Expert System that integrates a chain of process models, and predicts the sensitivity to statistical variation in the model inputs. The Expert System does not predict the sensitivity to observation errors or process model errors. Expertise that is not captured in the process models can be presented to the Expert System only through modification of the model inputs.

Goff⁷ proposed a statistical framework for mine burial modeling using Monte Carlo simulations. According to Goff's approach, input parameters for the Monte Carlo simulation are drawn separately from both uncertainty and variability probability density functions (pdf's). In his approach, the random nature of the problem is described through uncertainty, showing our lack of knowledge of "essentially deterministic factors", and natural variability in environmental parameters. Based on this differentiation, the author suggests finding (1) the percentage of mines buried and (2) the fraction of mines buried to certain extent.

Our work partially follows the Goff's idea by generating Gaussian distributions for the input parameters of the model (at mudline) based on measured data: vertical impact velocity (V_z), horizontal impact velocity (V_x), pitch angle at impact (q), angular rotation rate (\dot{q}), sediment layer density and sediment layer bearing strength. In our approach, the random nature of all the input parameters is treated equally, without separation into 'uncertainty' and 'variability'. It is argued that at present, there is no reliable way of distinguishing between these two concepts, implemented

using rather simplistic models of both, the material constitutive properties and the dynamics of penetrating mines.

Our study of mine impact burial is restricted to the sediment section only because of the original deterministic model's significant lack of accuracy in predicting the dynamic behavior of a mine in the water column⁵.

3. MONTE CARLO SIMULATION OVERVIEW

Any analytical method meant to imitate a real-life system is defined in the literature as a simulation. These methods are often used in situations when other analyses are too mathematically complex or too difficult to reproduce. A purely deterministic model will produce a single outcome, (usually the average scenario) when simulations are not used. In order to automatically analyze the effect of varying inputs of the modeled system on its outputs, various simulation techniques are used, with one of the most common being the Monte Carlo simulation. It generates the output values of uncertain variables by propagating the randomly sampled input parameter distributions through the model to simulate the behavior of a system.

The possible values of each uncertain variable, defined via its probability distribution functions (pdf's), may include Gaussian, uniform, lognormal or triangular distributions and should be selected based on the nature of the uncertain variables. In this paper, we use the Gaussian representations of all random variables in order to gain an insight into the stochastic performance of the predictive model.

4. MODEL DESCRIPTION

The current deterministic mine impact burial model describes the dynamics of a mine that can be deployed from the air or in water. The input data for that model are related to the mine kinematics parameters at release (vertical velocity, horizontal velocity, angle between the vertical and mine's long axis, and angular velocity), the mine's geometry and weight, altitude and medium in which released, water temperature and depth, sediment density and sediment bearing strength.

It was shown¹ that the current deterministic mine impact burial model's inadequate treatment of hydrodynamic effects in the water column is important source of errors. A solid object falling through fluid must comply with two fundamental physical principles, momentum balance and moment of momentum balance. The moment of momentum balance was not taken into consideration by the current model¹. Therefore, the probabilistic approach implemented in this work will analyze the mine impact burial for the sediment section only (as if the mine were released at the mudline), in order to avoid those errors induced by the model for mine motion in the water column. Experimental results⁶ from actual deployments of a full-size instrumented cylinder that can record its position and orientation in space will be used to determine the dynamic parameters on impact with the sediment, thus eliminating the influence of the highly inaccurate water-column prediction component.

The Monte Carlo algorithm has been employed in order to predict the mine impact burial. Given velocity and orientation data measured at the mudline for a full size, instrumented, mine-like cylinder⁶, we were able to generate probability density functions (pdf's) for the model's input parameters. Gaussian distributions have been considered for expressing the variability/uncertainty of the input parameters: vertical impact velocity (V_z), horizontal impact velocity (V_x), pitch angle at mudline (q), angular rotation rate (\dot{q}), sediment layer density and sediment layer bearing strength⁷. Physical properties of thirteen sediment layers were used to describe the sediment profile. The mean and standard deviation of these parameters are provided in Tables 1 and 2.

V_z [m/s]		V_x [m/s]		q [deg]		\dot{q} [rad/s]	
Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
4.208	1.154	-0.963	0.856	49.04	21.05	0.1468	0.3445

Table 1. Mean and Standard Deviation of the dynamic input parameters for instrumented cylinder.

			Bearing Strength [kPa]	
	Mean	Std. Dev.	Mean	Std. Dev.
Layer 1 (0-0.1 m)	1646.0	55.7	12	5
Layer 2 (0.1-0.2 m)	1788.4	56.2	32	14
Layer 3 (0.2-0.3 m)	1692.4	59.7	38	12
Layer 4 (0.3-0.4 m)	1671.4	73.4	40	10
Layer 5 (0.4-0.6 m)	1654.3	43.9	47	13
Layer 6 (0.6-0.8 m)	1693.5	69.2	52	15
Layer 7 (0.8-1.0 m)	1746.5	97.8	59	21
Layer 8 (1.0-1.2 m)	1708.8	118.8	59	13
Layer 9 (1.2-1.4 m)	1819.0	98.2	57	16
Layer 10 (1.4-1.6 m)	1754.0	101.4	73	32
Layer 11 (1.6-1.8 m)	1757.3	79.7	53	11
Layer 12 (1.8-2.0 m)	1757.3	79.7	53	17
Layer 13 (2.0-5.0 m)	1757.3	79.7	53	17

Table 2. Mean and Standard Deviation of the sediment layers' wet density and bearing strength

A random sampling of the Gaussian generated distributions has been performed in order to feed the model with input data. The results of 1000 simulations were recorded and provide the basis for frequency histograms. An adequate number of samples used in this analysis was decided upon based on a sensitivity study. It was found that the sampling at any higher rate, in excess of 1000, results in only marginal added value in the accuracy of the output distributions.

Given a critical percentage of the mine surface area buried, above which the countermeasure force cannot detect the mines reliably, additional analysis of the stochastic model output can be done. The probability of not detecting (or the risk of not detecting) buried mines can be computed

by integrating the relative histogram of the percentage of surface area buried over the following range:

$$\text{Percentage_Surface_Area_Buried} > \text{Percentage_Surface_Area_Buried}_{\text{critical}}$$

These values can be normalized by dividing the number of mines within this range by the total number of runs, as illustrated in Table 3.

5. MODEL IMPLEMENTATION

The probabilistic approach to the impact mine burial model was implemented in MATLAB and the model's output for 1000 runs was used for generating the output frequency histograms. The histograms of the percentage of the mine's surface area buried, mine burial depth and pitch angle (at rest in the seabed) are shown in Figures 1 thru 3.

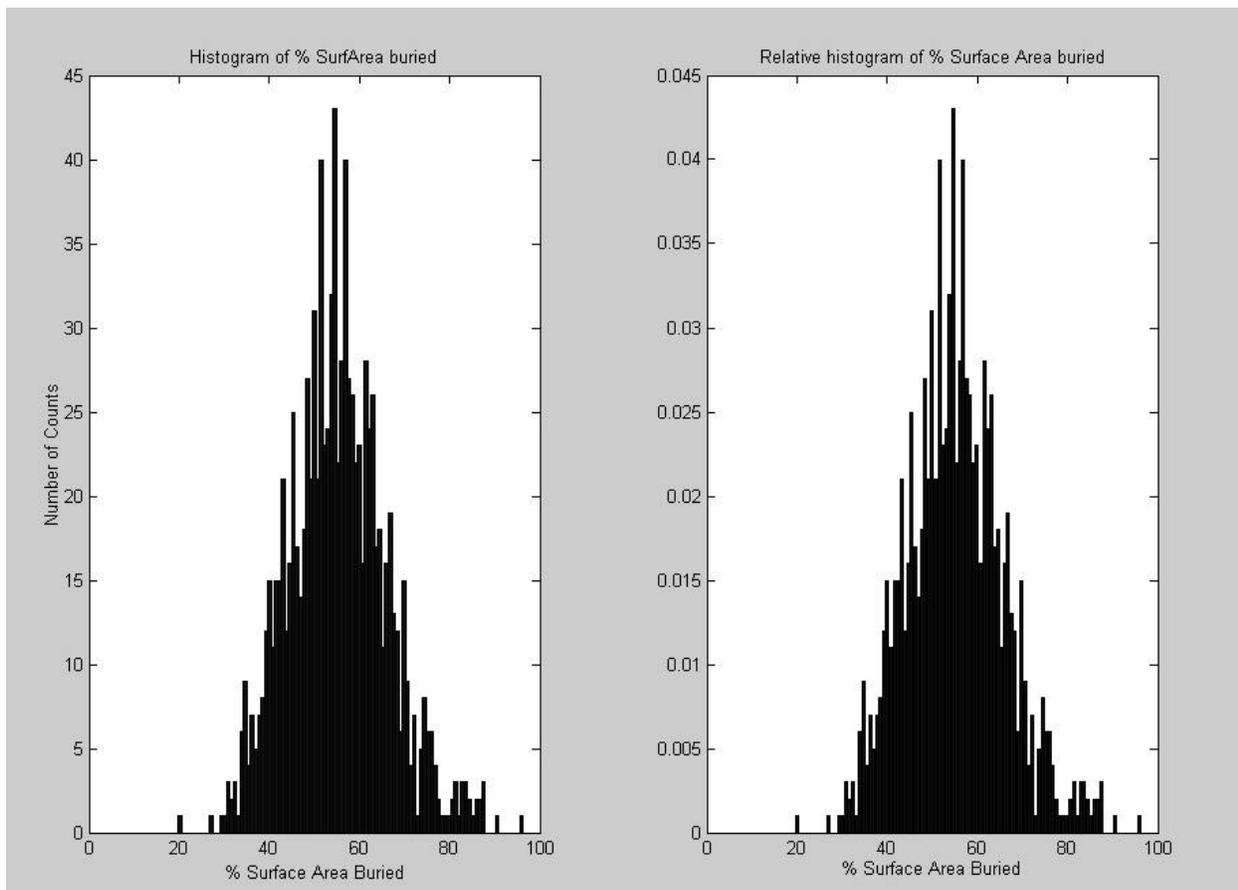


Figure 1. Histograms of Percentage of Mine Surface Area buried

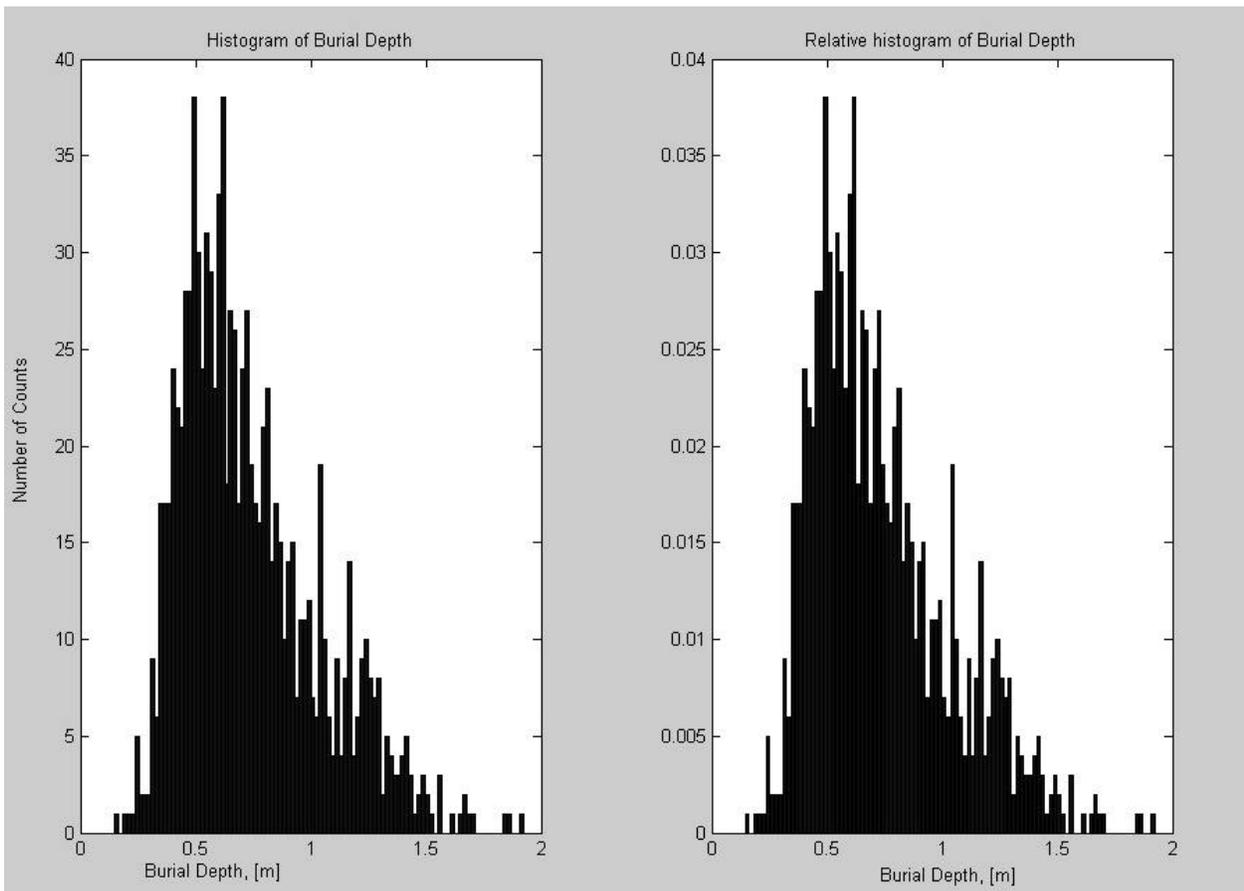


Figure 2. Mine Burial Depth histograms

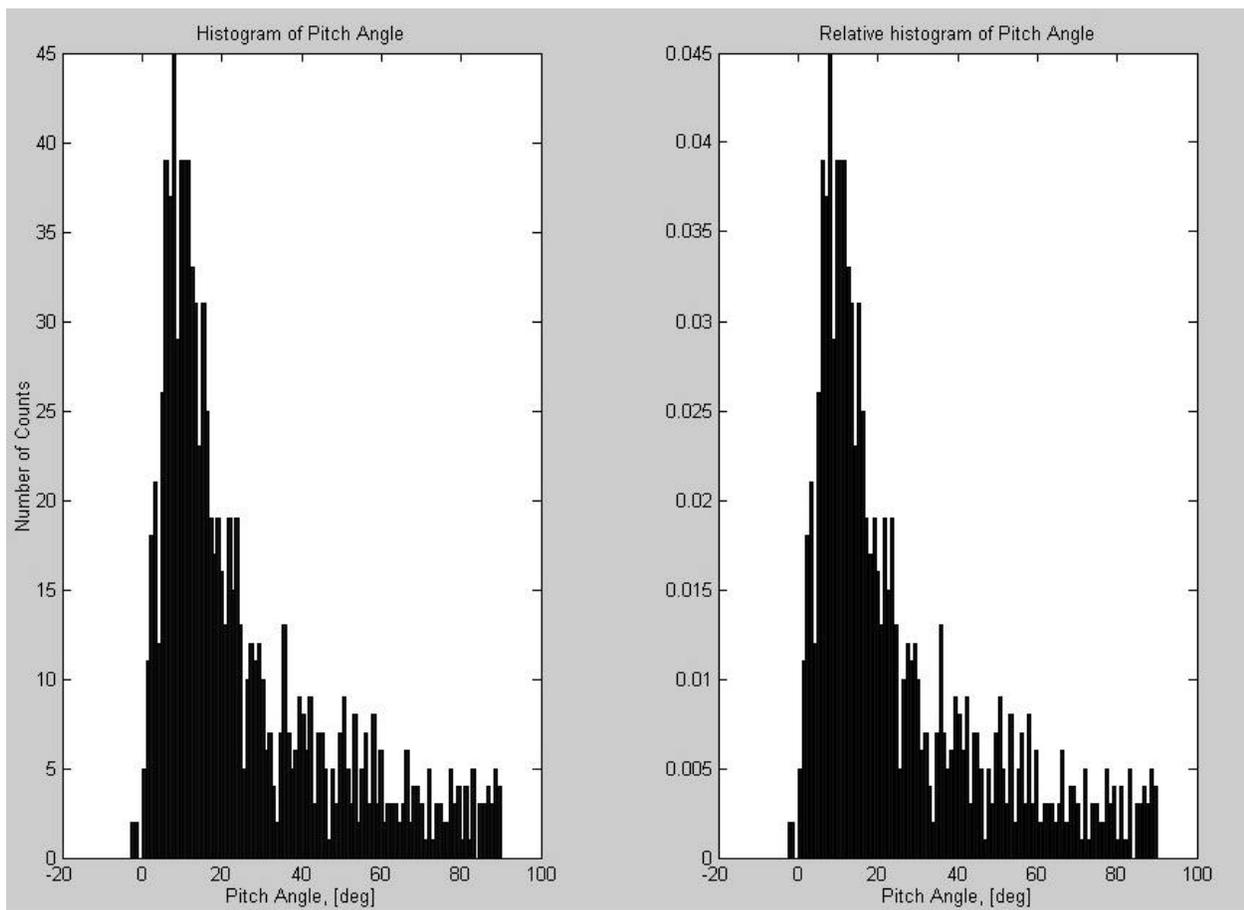


Figure 3. Pitch angle (at rest in the seabed) histograms

The experimental data collected by divers, albeit limited, compares well with the probability density functions of the model's output variables.

	Critical Limits of Percentage Surface Area Buried		
	10 %	20 %	75 %
Probability of not detecting buried mines	1.000	0.999	0.042

Table 3. The probability of not detecting (or the risk of not detecting) buried mines for various critical levels

Based on several separate simulations, a probability chart of not detecting mines buried more than a predefined critical limit can be achieved. An example of such a probability chart for a specific mine geometry and a particular location is shown in Figure 4. The error-bars indicate the variability in the output probability as a result of six realizations, indicating the accuracy of the selected number of samples used in our calculations.

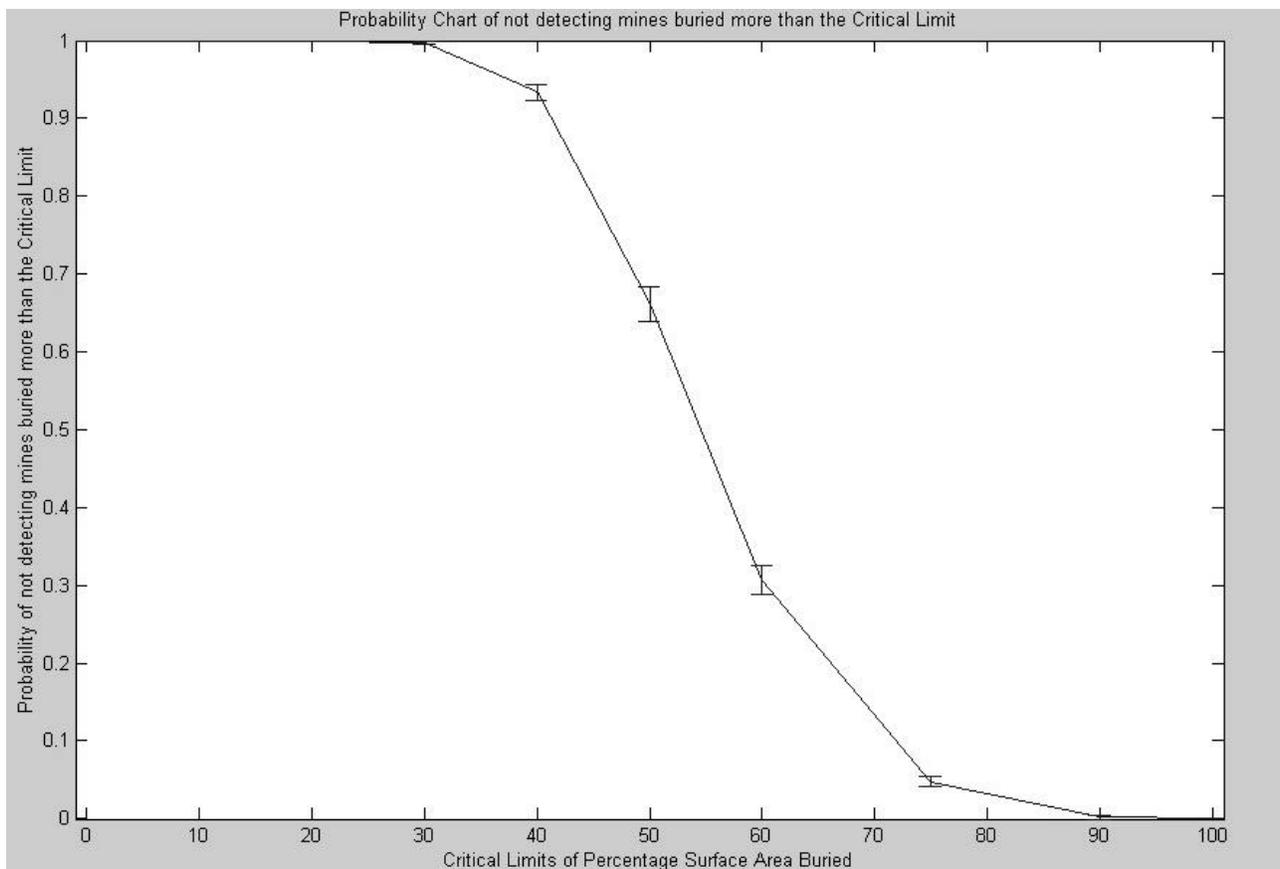


Figure 4. Probability Chart of not detecting mines buried more than an MCM forces' predefined critical limit

6. SUMMARY AND CONCLUSIONS

In this work, we proposed and implemented a probabilistic approach of the mine impact burial model in order to incorporate the uncertainties and variability of the seafloor sediment parameters and of the mine dynamics on contact with the seafloor. Since the original deterministic model did not correctly consider the hydrodynamic effects in the water column, our study of mine impact burial was focused only on the sediment section in order to avoid induced errors. A Monte Carlo method was implemented to study the overall probability distributions of the output parameters. The results produce compare well with the experimentally observed distributions.

An example of a risk analysis based on the probability of encountering an undetectable bottom mine with a specific geometry and in a specific location is performed and illustrated in a probability chart. This approach is suggested to be a useful tool for the MCM decision process.

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REFERENCES

1. Dawson, Cutler VADM, and VADM Mike Bucchi, Naval Message, MSGID/GENADMIN/COMSECONDFLT/J9/FEB//Subj: Sea Power-21 Implementation Message NR-3; Operational Agent Required Warfighting Capabilities List, from COMSECONDFLT, dated 21 Feb 03.
2. Bucchi, Mike VADM, and VADM Mike Mullen, "Sea Shield: Projecting Global Defensive Assurance, Sea Power 21 Series—Part II," *Naval Institute Proceedings*, Nov 2002, at <http://www.usni.org/Proceedings/Articles02/PRObucchi11.htm>
3. P. C. Chu, T. B. Smith, and S. D. Haeger, "Mine Impact Burial Prediction Experiment", *The Journal of Counter-Ordnance Technology*, 2002
4. R. A. Arnone, and L. E. Bowen, "Prediction model of the time history penetration of a cylinder through the air-water-sediment phases". NCSC TN 734-36. Naval Coastal Systems Center, Panama City, FL, 1980.
5. L. J. Satkowiak, "User's guide for the modified impact burial prediction model". NCSC TN 884-87. Naval Coastal Systems Center, Panama City, FL, 1987.

6. P. J. Mulhern, "Experiments on mine burial on impact-Sydney Harbour". *U.S. Navy J. of Underwater Acoustics*, 43, 1271-1281, 1992.
7. P. C. Chu, V. Taber, and S. D. Haeger, "A mine impact burial model sensitivity study", Institute of Joint Warfare Analysis, Naval Postgraduate School, Technical Report, NPS-IJWA-00-003, pp. 48, 2000
8. P.J. Valent, K.T. Holland, A.W. Green, S. Theophanis, C. King, M.D. Richardson, G.R. Bower, P. Congedo and W. Lewis, "Observations of Velocities and Orientations of Cylindrical Bodies at Terminal Condition in Water," *Proc. of the Fifth International Symposium on Technology and the Mine Problem, NPS, Monterey*, April 21-25, 2002
9. A.V. Abelev, P.J. Valent, N.G. Plant and K.T. Holland, "Evaluation and Quantification of Randomness in Free-Fall Trajectories of Instrumented Cylinders," *Proceedings OCEANS 2003 MTS/IEEE*, in CD-ROM, San Diego, CA, 22-26 Sep 2003.
10. S. Rennie and A. Brandt, "An expert system approach for predicting mine burial", *Proc. of the Fifth International Symposium on Technology and the Mine Problem, NPS, Monterey*, April 21-25, 2002
11. J. A. Goff, "A statistical framework for Monte Carlo mine burial modeling experiments", A technical report for the ONR Mine Burial program, April 2002.
12. R. B. Hurst, "Mine Impact Burial Prediction Model – Technical Description of Recent Changes and Developments (U)", Report 149, Defence Scientific Establishment, Auckland, New Zealand, 1992